$S = \bigoplus_{d \in \mathbb{Z}_{\geq 0}} S_d$ graded ring$/k$ generated in degree one
i.e. $k[S_1] \twoheadrightarrow S$

$\mathrm{Proj}(S)$ scheme with affine opens
$$D_f = \mathrm{Spec}((S_f)_0) \quad \text{for } f \in S_1$$
and gluing maps $D_f \cap D_g \xrightarrow{\sim} D_g \cap D_f$ $\quad f, g \in S_1$
given by canonical isomorphisms
$$(S_f)_{0,g} \cong (S_{fg})_0 \cong (S_g)_{0,f}.$$

$M = \bigoplus_{d \in \mathbb{Z}} M_d$ graded module over S

$\widetilde{M}$ quasicoherent sheaf on $\mathrm{Proj}(S)$ with
$$\Gamma(D_f, \widetilde{M}) = (M_f)_0$$

Serre twist : $M(m) = \bigoplus_{d \in \mathbb{Z}} M_{d+m}$

$\Rightarrow \Gamma(D_f, \widetilde{M(m)}) = (M_f)_m$

Example: projective space $\mathbb{P}^n = \mathrm{Proj}(\underbrace{k[x_0, \dots, x_n]}_{= S})$

$U_i = D_{x_i}$
$$(S_{x_i})_0 = k\left[\frac{x_0}{x_i}, \dots, \frac{x_n}{x_i}\right]$$
$\mathcal{O}_{\mathbb{P}^n}(m) = \widetilde{S(m)}$

$$\Rightarrow \Gamma(U_i, \mathcal{O}_{\mathbb{P}^n}(m)) = (S_{x_i})_m$$
$$= \langle x_0^{a_0} \cdots x_i^{b} \cdots x_n^{a_n} \mid a_j \geq 0, b \in \mathbb{Z}, b + \sum_{j \neq i} a_j = m \rangle$$
$$\cong x_i^m \, k\left[\frac{x_0}{x_i}, \dots, \frac{x_n}{x_i}\right] = x_i^m \mathcal{O}_{U_i}$$

Transition functions:

$$x_i^m\, k\left[\frac{x_0}{x_i},\dots,\frac{x_n}{x_i}\right]\left[\left(\frac{x_j}{x_i}\right)^{-1}\right] \longrightarrow x_j^m\, k\left[\frac{x_0}{x_j},\dots,\frac{x_n}{x_j}\right]\left[\left(\frac{x_i}{x_j}\right)^{-1}\right]$$

$$x_i^m f\left(\frac{x_0}{x_i},\dots,\frac{x_j}{x_i},\dots,\frac{x_n}{x_i}\right) \longmapsto x_j^m \left(\frac{x_i}{x_j}\right)^m f\left(\frac{x_0}{x_j}\left(\frac{x_i}{x_j}\right)^{-1},\dots,\underbrace{\left(\frac{x_i}{x_j}\right)^{-1}}_{j\text{th}},\dots,\frac{x_n}{x_j}\left(\frac{x_i}{x_j}\right)^{-1}\right)$$

In the computation of $\check{H}_j\left(\mathbb{P}^n, \bigoplus_{m\in\mathbb{Z}} \mathcal{O}(m)\right)$ we crucially used the fact.

$$\boxed{H^0\left(U_i, \bigoplus_{m\in\mathbb{Z}} \mathcal{O}(m)\right) = \bigoplus_{m\in\mathbb{Z}} x_i^m \mathcal{O}_{U_i}}$$

Sanity check: Understand the description

$$\Gamma(U_i, \mathcal{O}(m)) = x_i^m \mathcal{O}_{U_i}$$

in terms of the line bundle associated to the cartier divisor $(U_j, 1), (U_i, x_i^{-m})$ or to the Weil divisor $m H_i$